

ADDENDUM

In the article *Gen. Rel. Grav.* **32**, 1633 (2000), by J. G. Pereira and C. M. Zhang (here referred as Paper I), the special relativity energy-momentum tensor was used to discuss the neutrino phase-splitting in a weak gravitational field. However, it would be more appropriate to use the general relativity energy-momentum tensor. When we do that, as we are going to see, some results change, but the basic conclusion remains the same.

The ratio between the two terms in the right-hand side of Eq.(6) of Paper I can be written as

$$\xi = \frac{|h_{\mu\nu}dx^\mu dx^\nu|}{(ds^o)^2} = \frac{\alpha\delta_{\mu\nu}dx^\mu dx^\nu}{(ds^o)^2} = \frac{\alpha\delta_{\mu\nu}dx^\mu dx^\nu}{ds^2} \frac{ds^2}{(ds^o)^2}. \quad (1)$$

As

$$\frac{ds}{ds^o} = \sqrt{1 - \xi}, \quad (2)$$

we have consequently

$$\xi = \frac{\alpha}{m^2} (1 - \xi) [(P^o)^2 + (P^r)^2], \quad (3)$$

where the momentum is defined by $P^\mu = m(dx^\mu/ds)$, with $(P^r)^2 = (P^x)^2 + (P^y)^2 + (P^z)^2$. Therefore, we get

$$\xi = \frac{\alpha(2\gamma^2 - 1)}{1 + \alpha(2\gamma^2 - 1)}, \quad (4)$$

where $\gamma = (P^o/m)$ is the relativistic factor. The approximate mass-shell condition $(P^o)^2 - (P^r)^2 \approx m^2$ has been used in the above expression.

The possible values of ξ is restricted to the range $0 \leq \xi \leq 1$, where the values 0 and 1 represent respectively the vacuum and the null cases. In the absence of gravitational field, which corresponds to $\alpha \rightarrow 0$, we find $\xi = 0$, and the vacuum situation is recovered. When $v \rightarrow c$, which corresponds to an ultra relativistic case, $\gamma \rightarrow \infty$, and we obtain $\xi = 1$. We see from Eq.(4), therefore, that in fact $0 \leq \xi \leq 1$ in any situation.

We can now discuss the expansion conditions. For a low-energy object, as for example a thermal neutron in the laboratory whose typical velocity is $v^2 \sim 10^{-10}$, we have $\gamma \simeq 1$, and consequently

$$\xi = \frac{\alpha}{1 + \alpha} \simeq \frac{r_s}{r} \ll 1. \quad (5)$$

This is the conventional weak-field condition, which means that the phase splitting can be performed. On the other hand, for ultra-relativistic massive neutrinos, the relativistic factor is $\gamma^2 \sim 10^{12}$, and in the case of the Earth gravitational potential, for which $(r_s/r) \sim 10^{-11}$, we get $\xi = 0.95 \approx 1$. Consequently, for these particles, the phase splitting represented by Eq.(2) of Paper I cannot be performed. Therefore, when the general relativity energy-momentum tensor is used, despite some numerical differences in the values assumed by ξ , the basic conclusion of Paper I remains valid.

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